THE MECHANICAL PRE-STRESSING IN ULTRASONIC PIEZOTRANSDUCERS

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Ultrasonic motors have been investigated for several years and have already found their first practical applications. Their key feature is that they are able to produce a high thrust force related to their volume. In different applications if the level of pre-stressing is low the parameters of the piezoelectric ceramics are not altered and the shifting on frequencies are attributed to coupling between rotor and stator. In this paper the authors try to describes a mathematical model to explain the contact mechanism at the interfaced between the transducers parts, to evaluate this effect based on difference of cross-section between piezoelectric ceramics under pre-stressing. The measurements show a proportional relation between the mechanical pre-stressing and effective coupling of the piezoelectric ceramics.

Keywords: ultrasonic motor, piezoelectric, pre-stressing.

1. INTRODUCTION

The mechanical pre-stressing produces shifting on resonance and anti-resonance frequencies on ultrasonic motors. Two possible causes for these shifting have been studied: changes of the physical characteristic parameters or changes on the coupling of the active piezoelectric pieces. The physical properties of different ceramics do not change significantly in the low levels of pre-stressing, up to 70 MPa, so another possible reason to explain the shifting frequencies is their dependence on coupling between active elements of piezotransducer under the pre-stressing force. This paper presents a mathematical model of electric impedance, that takes into account the difference of cross-sections between active elements on interfaces. The curves from experimental data are compared with ones from theoretical model and the dependence between mechanical pre-stressing and effectiveness of matching on interfaces is verified.

2. THE PIEZOTRANSDUCER MODEL

The modelling presents the electric impedance expression of the transducers used in experiments. This expression shows the coefficients of the piezoelectric equations and allows the identification of resonance and anti-resonance frequencies. The piezoceramics and the aluminum pieces that compose the transducers are cylindrical elements with a central hole where there is a bolt that tightens the assembly. The transducers are axisymmetric, making modelling simpler and allowing to analyse the problem with one half of the transducer. Fig. 1 shows the transducer design and the part considered in modelling. The model takes into account the following assumptions: the radial dimensions of the transducer are higher than wavelength; the transducer pieces are loss-free; the transducer vibrates along its longitudinal axis so that just longitudinal waves are regarded, the radial modes are neglected; and the interference of electrodes and the bolt is neglected.

The modelling is based on waves (1) and piezoelectric equations (2) and (3):

\[ \rho \frac{\partial^2 u}{\partial t^2} = \varepsilon_{33} \frac{\partial^2 D_3}{\partial x_3^2} \]

\[ T_3 = \varepsilon_3^{(0)} S_3 - \kappa_{33} D_3 \]
where: \(q\) (kg/m\(^3\)), the density from propagation medium; \(c_{33}^D\) (N/m\(^2\)), the elastic coefficient of the ceramic; \(u_3\) (m), the displacement of the particles through propagation medium; \(x_3\) (m), the position along of the propagation axis; \(t\) (s), the time; \(D_3\) (C/m\(^2\)), the electric displacement on 3-axis; \(S_3\) (m/m), the strain on 3-axis; \(T_3\) (N/m\(^2\)), the stress on 3-axis; \(E_3\) (N/C), the electric field on 3-axis; \(h_{33}\) (C/N), the piezoelectric coefficient; \(b_{33}\) (m/F), the inverse of the electric permittivity of the clamped ceramic.

The spatial part of the solutions of wave equation in piezoelectric ceramics are:

\[ u_2 = A \sin(px_3) + B \cos(px_3) \]  
\[ T_2 = p c_{33}^D [A \cos(px_3) - B \sin(px_3)] - h_{33} D_3 \]

And in the aluminum pieces are

\[ u_{2A} = A_1 \sin(qx_3) + B_1 \cos(qx_3) \]  
\[ T_{2A} = q Y_A [A_1 \cos(qx_3) - B_1 \sin(qx_3)] \]

where \(\omega\) (rad/s), the angular frequency; \(v_3\) = \(\sqrt{Y_A / \rho_A}\) (m/s), the propagation speed of longitudinal waves in aluminum; \(Y_A\) (N/m\(^2\)), the aluminum Young’s modulus; and \(A\), \(B\), \(A_1\) and \(B_1\), constants determined by continuity conditions.

The modelling of multilayer transducers involves continuity conditions. In this case, the variations of cross-sections impose the stress and the flux volume continuity in each interface of the transducer and are given by:

\[ u_2(0) = 0 \]  
\[ s_{12} u_2(a) = u_{2A}(a) \]  
\[ T_2(a) = T_{2A}(a) \]  
\[ T_{5A}(b) = 0 \]

where \(a\) is the thickness of each ceramic; \(b\), the length of each half of the transducer; and \(s_{12}\), the ratio of cross-section between piezoelectric ceramic and aluminum piece.
In the central position of the axisymmetrical transducers, the displacement is null. Eqs. (9) and (10) determine the continuity of volume and stress on interface piezoceramic-aluminum and Eq. (11) means that the end of transducer is free.

A system of equations is obtained by substituting the solutions (4)-(7) on the continuity conditions (8)-(11). By solving this system, $E_3$ and $D_3$ using Eqs. (2) and (3) can be obtained, and the voltage, $V = \int E_3 d x_2$, and the electric current, $i = \frac{dD_3}{dt}$ can be derived. The modulus electric impedance (given in $\Omega$) is calculated by $Z = \frac{V}{i}$, so that:

$$|Z| = \frac{1}{\omega C_0} \left( 1 - \frac{h_{33}^2 s_{12}}{\beta_{2s}^2 a} pc_{33}^2 s_{12} \alpha \left( \alpha \right) \right)$$

where, $\alpha_1 = \cos(qa) + \sin(qa) \tan(qb)$ and $\alpha_2 = \sin(qa) - \cos(qa) \tan(qb)$

3. EXPERIMENTAL RESULTS

The minimum and maximum values from impedance function determines the resonance and anti-resonance frequencies for each value of $s_{12}$. By plotting the resonances and anti-resonances as function of cross-section ratio, it is possible to compare the shapes of these plots with the experimental ones and to determine a phenomenological relation between the mechanical pre-stressing and the cross-section ratio.

The transducers, named T1 and T2 have in each end a aluminum piece with 10 and 30 mm of length, respectively. The characteristic parameters of the ceramics are $c^{D_{33}} = 13.9 \times 10^{10}$ N/m$^2$, $\varepsilon^{S_{33}} = 11$ nF/m, $h_{33} = 14.8 \times 10^8$ C/N. The cross-section ratio is calculated by using the resonance frequency obtained experimentally on the model when $Z = 0$ in Eq (12) and solving it through a numerical method.

The experimental and the simulation results are plotted in Figs. 2 and 3. The curves present the resonance and anti-resonance frequencies as a function of experimental mechanical pre-stressing and as a function of transversal section ratio. Figs. 4 - 5 show the mechanical pre-stressing as a function of cross-section ratio. These graphics result from substituting the resonance frequencies obtained experimentally in the mathematical model when $|Z| = 0$.

Figure 2: Resonance (lower curve) and anti-resonance (upper curve) frequencies as a function of mechanical pre-stressing and ratio of cross-section for T1 piezoelectric transducer.
4. CONCLUSIONS

In Figs. 2 and 3, the shapes of the resonance and antiresonance curves as a function of mechanical pre-stressing are similar to ones relative to cross-section ratio obtained through simulation. The frequencies verified experimentally are close to those obtained in the simulation when $s_{12}$ is higher than 0.4. This means that $s_{12}$ rises proportionally to mechanical pre-stressing increase, and yields improvement on acoustic coupling of the active elements of the transducer. In spite of the highest frequencies in experimental and the simulated results are close, there are some aspects that deserve to be regarded:

- The effect of the central bolt is neglected in the mathematical model, and this presence contributes to the increase of the resonances, because it is made of steel with a higher mechanical impedance; aluminum pieces and steel bolt compose the mechanical charge connected to piezoelectric ceramics. If the thickness of aluminum pieces increases, the effect of bolt on resonance can be neglected, as done in this study, because the experimental and the simulated frequencies are closed.

- The resonance frequencies used on simulations to obtain the curves on Figs. 4 and 5 are provided of dynamical measurements, while in the modelling the coefficients were derived from static conditions. The model takes into account a simpler piezoelectric transducer, where losses and other vibration modes are despised and the differences between theoretical and experimental results can occur.

![Figure 3: Resonance (lower curve) and anti-resonance (upper curve) frequencies as a function of mechanical pre-stressing and ratio of cross-section for T2 piezoelectric transducer.](image3)

![Figure 4: Ratio of cross-section as function of mechanical pre-stressing on T1](image4)
The mechanical pre-stressing in ultrasonic piezotransducers

Figure 5: Ratio of cross-section as function of mechanical pre-stressing on $T_2$.

In conclusion if the mechanical pre-stressing is higher up to 50 MPa, the ratio $s_{12}$ is approximately equal to 1, this means that around this value the acoustic coupling between the pieces is satisfactory.

REFERENCES