The robots application extends continually to various domains as medicine, housekeeping, extraterrestrial research, underwater applications, inspection tasks in difficult of access areas a.s.o. The manipulators with redundant structures represent the perfect choice for numerous applications. The biped autonomous robots that cooperate or not are an edifying example for the ramified redundant structures use. The anthropomorphic structures with many degrees of freedom reproduce the human skeleton or only a part of it.

Keywords: kinematics, planar manipulator, redundancy, ramified structure, cooperative arms.

1. INTRODUCTION

The advantages of the redundant structures concerning the manipulators used in several applications are obvious. The manipulability, dexterity, versatility, the possibility for the optimisation of different performance criteria, there are only ones of them. The redundant structures inspired from the human skeleton are ramified open or serial-parallel structures. The kinematical solving of these structures uses the same mathematical relations that are applied for the redundant serial structures.

The authors of this paper use a strategy of “sectioning” (separating) the parallel branches and their “recomposition” in the initial structure. A solving method consists of adapting the Jacobian matrix by its fictive application to a special part of the structure. The angular variations in the structure part, where the unchanged Jacobian matrix is applied, depend on the angles of the structure part where this matrix is not applied, respecting its global character; these angles are known from the previous step. In this way the initial kinematic chain recomposition becomes simple. Another solving method consists in the inverse kinematic run through a part of the structure, as one of the branches. This method is computational expensive and needs additional mathematical relations.

It is easy to observe that in the point of structure ramification there is a double joint, which represents the point of structure “sectioning” or “recomposition”.

The direct geometric model gives the relation between the end-effector position and the joint coordinates:

\[ x = f(q); \quad x = [x_1, x_2, \ldots, x_m]^T; \quad q = [q_1, q_2, \ldots, q_n]^T \]  

where \( n \) is the number of degrees of freedom (d.o.f.) and \( m \) is the space dimension. The degree of redundancy of the structure is \( n - m \).

If one differentiates the direct geometric model, it follows that:

\[ \dot{x} = \frac{\partial f(q)}{\partial q} \dot{q} \quad ; \quad \frac{\partial f(q)}{\partial q} = J(q) \]  

The direct differential model is:

\[ \dot{x} = J(q) \dot{q} \quad \text{or} \quad \delta x = J(q) \delta q \]
where $\delta x = [\delta x_1 \ \delta x_2 \ldots \ \delta x_n]^T$, $\delta q = [\delta q_1 \ \delta q_2 \ldots \ \delta q_n]^T$ and $J$ is the Jacobian matrix of the analysed structure.

In redundancy case, $n > m$, the relation between final-effector velocities and joint-velocities becomes:

$$\delta q = J^+ \delta x$$

(4)

The relation (4) represents the inverse differential kinematic model; $J^+$ is a non-square matrix with dimensions $(n \times m)$, called the pseudoinverse of the Jacobian matrix. This pseudoinverse matrix allows constraint function or performance criteria introduction in the null space of the Jacobian matrix, for proposed objective accomplishment by inverse differential kinematic model extension.

The Jacobian matrix has the following form:

$$J = \begin{bmatrix}
\frac{\partial x_E}{\partial \theta_1} & \ldots \\
\frac{\partial x_E}{\partial \theta_i} & \ldots \\
\frac{\partial x_E}{\partial \theta_n} & \ldots
\end{bmatrix} ; \ i = 1, \ldots, n$$

(5)

with dimensions $J \in \mathbb{R}^{m \times n}$.

The pseudoinverse matrix $J^+$ is defined using following relation:

$$J^+ = J^T (J J^T)^{-1}$$

(6)

with dimensions $J^+ \in \mathbb{R}^{n \times m}$.

2. PROBLEM FORMULATION

The figure 1 shows a redundant planar manipulator with ten d.o.f. having a serial-parallel (ramified) open structure composed from elements with lengths $l_1 \ldots l_4$ that forms a positioning structure and two ramifications $l_5 \ldots l_7$, respectively $l_5' \ldots l_7'$, the upper and lower ramifications that are called orientation structures.

The degree of redundancy $n - m$ is different, depending on the number of joints that are considered in the Jacobian matrix composition for different structure sections.

Considering the number of joints, we can anticipate that the structure has ten d.o.f. Without analysing in this paper the d.o.f. for the proposed kinematic structure, we can anticipate related d.o.f. between those that determine the positioning structure and those that determine both branches of the structure.

The branches of the proposed kinematic structure from the figure 1 are cooperative, so that they will cooperate for the segment $E_1E_2$ manipulation in $x_1Ox_2$ plane concluding, for example, a roto-translation of this segment.

![Figure 1 – Ramified structure with ten d.o.f.](image-url)
The initial position of the structure is:

\[
l_1 = l_2 = l_3 = l_4 = 4; \quad l_5 = l_6 = l_6' = l_6'' = 3; \quad l_7 = l_7' = 1.9
\]

\[
\Theta_i = [15^\circ \ 60^\circ \ 90^\circ \ -60^\circ \ -60^\circ \ 90^\circ \ 30^\circ]^T
\]

\[
\alpha_i = [15^\circ \ 60^\circ \ 90^\circ \ -60^\circ \ 30^\circ \ -120^\circ \ -60^\circ \ -60^\circ \ -90^\circ \ -120^\circ \ -60^\circ \ -60^\circ \ 60^\circ \ 30^\circ \ -60^\circ \ -120^\circ ]^T
\]

with \( \alpha_i = 0 \), for \( i = 1 \ldots 4 \).

The direct geometric models for \( l_1 \ldots l_7 \) or \( l_1 \ldots l_4 \ l_5' \ldots l_7' \) courses are:

\[
x_{1E1} = l_1 S_1 + l_2 S_{12} + \ldots + l_7 S_{1234567}
\]

\[
x_{2E1} = l_1 C_1 + l_2 C_{12} + \ldots + l_7 C_{1234567}
\]

For course \( l_1 \ldots l_4 \ l_5' \ldots l_7' \): \( \alpha_i = \theta_i \) for \( i = 1 \ldots 4 \), \( \alpha_i \neq \theta_i \) for \( i = 5 \ldots 7 \) and \( l_i = l_i' \) for \( i = 5 \ldots 7 \).

In the relations (8), the notations have the following signification: \( \sin \theta_1 = S_1 \), \( \cos \theta_1 = C_1 \), \( \sin(\theta_1 + \theta_2) = S_{12} \) etc.

### 3. PROPOSED STRATEGY

For the kinematic solving of the structure shown in the figure 1, it is proposed the “sectioning” of the double joint \( 4 \equiv 4' \), as it is shown in the figure 2.

![Figure 2 – Decoupling of the double joint 4.](image)

Applying the inverse differential kinematic model (relation (4)) on the kinematic chains \( l_1 \ldots l_7 \) and \( l_1 \ldots l_4 \ l_5' \ldots l_7' \), for the imposed velocities \( \delta x_{E1} \) and \( \delta x_{E2} \) of the points \( E_1 \) and \( E_2 \), the joints 4 and 4' will replace the positions shown in the figure 2. On the segment \( 44' \), a correction position \( 4_c \) is chosen, which can be, in this case, the middle point between the points 4 and 4', whose coordinates are calculated using the direct geometric model (relation (1)).

After the coordinates of the correlative position \( 4_c \) are determined, the joint 4 or 4' is brought in the correlative position. This is done by imposing the joint 4 or 4' to have the necessary coordinate variation by generation in more steps or in one step if the coordinate variation is small.

The recoupling of the partial kinematic chains is accomplished using a method proposed by authors and presented in the following.

### 4. METHOD OF STRUCTURE “SECTIONING” AND PARTIAL KINEMATIC “STIFFENING”

One eliminates the lower branch \( l_5' \ldots l_7' \) from the structure presented in the figure 1. The remaining part is shown in the figure 3.
The “sectioning” of the structure can be realised in every interior joint 1 . . . 6. Choosing this joint being 4, one obtains the kinematic chains $l_1 . . . l_4$ and $l_5 . . . l_7$ with the angles $\theta_1 . . . \theta_4$ and $\theta_5 . . . \theta_7$ respectively.

The direct differential kinematic model (relation (3)) can be enhanced and partitioned:

$$
\begin{bmatrix}
\delta x_1 \\
\delta x_2
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \cdots & \frac{\partial x_1}{\partial q_r} & \cdots & \frac{\partial x_1}{\partial q_{r+1}} & \cdots & \frac{\partial x_1}{\partial q_n} \\
\frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} & \cdots & \frac{\partial x_2}{\partial q_r} & \cdots & \frac{\partial x_2}{\partial q_{r+1}} & \cdots & \frac{\partial x_2}{\partial q_n}
\end{bmatrix}
\begin{bmatrix}
\delta q_1 \\
\delta q_r \\
\delta q_{r+1} \\
\delta q_n
\end{bmatrix} =
\begin{bmatrix}
J_{1r} & J_{1m} \\
J_{2r} & J_{2m}
\end{bmatrix}
\begin{bmatrix}
Q_r \\
Q_m
\end{bmatrix} \tag{9}
$$

where:
- $J_{1r} = 0$ represents kinematic influence annulment (“stiffening”) for joints 1 . . . r on Ox1 axis;
- $J_{2r} = 0$ represents kinematic influence annulment (“stiffening”) for joints 1 . . . r on Ox2 axis;
- $J_{1m}$ represents kinematic influence (mobility) for joints \(r+1 . . . n\) on Ox1 axis;
- $J_{2m}$ represents kinematic influence (mobility) for joints \(r+1 . . . n\) on Ox2 axis;
- $Q_r = [\delta q_1, \delta q_2, \ldots, \delta q_r]^T$ represents angular velocities of the “stiffened” joints;
- $Q_m = [\delta q_{r+1}, \delta q_{r+2}, \ldots, \delta q_n]^T$ represents angular velocities of the “non-stiffened” joints.

One notices that:
- for $r = 0$ one obtains the relation (3); the entire kinematic chain is mobile;
- for $r = n$ one obtains the “stiffening” of the entire kinematic chain;
- for $0 < r < n$ one obtains the intermediate situations;
- the joint degree of freedom can be “designed” on axes in a participative influence manner;
- the relation (9) can be easily adapted for spatial applications;
- for the proposed application in this paper, $r = 4$ and $n = 7$.

5. METHOD APPLICATION ALGORITHM

The proposed algorithm is composed of following main steps:

A1 – the velocity $\delta x_{E1}$ is imposed to the point $E_1$ and the inverse differential kinematic model is applied on the course $l_1 . . . l_2$; the joint-angles $\theta_i, i = 1 . . . 7$ are obtained. The joint 4 occupies the position shown in figure 2.

A2 – the velocity $\delta x_{E2}$ is imposed to the point $E_2$ and the inverse differential kinematic model is applied on the course $l_1 . . . l_4 l_5' . . . l_7$; the joint-angles $\theta_i, i = 1 . . . 7$ are obtained. The joint 4 occupies the position $4'$ shown in figure 2.

A3 – using the determined coordinates of the points 4 and 4' from figure 2, the coordinates of the correction point position $4_c$ are calculated.
A4 – applying the relation (4) on the course \( l_1 \ldots l_4 \) with the initial angles \( \theta_i, i = 1 \ldots 4 \) (or \( \alpha_i, i = 1 \ldots 4 \)) and considering the coordinates variations during the displacement 4→4c (or 4→4a), the final values for the angles \( \theta_i \) or \( \alpha_i, i = 1 \ldots 4 \) are obtained. In the following, it is assumed that the angles \( \theta_i, i = 1 \ldots 4 \) are known.

A5 – the angles \( \theta_i, i = 1 \ldots 4 \), obtained at the A4 step, and to the angles \( \theta_i, i = 5 \ldots 7 \), obtained at the A1 step, determine the position of the point E_i. Because the elements 1 \ldots 4 changed the orientations from the A1 step, the elements 5, 6 and 7 become quasi-parallel with the initial orientations.

A6 – the velocity \( \delta x_{E_1} \) is imposed again to the point E_1 (as in the A1 step) and the angles \( \theta_i, i = 1 \ldots 7 \) are determined applying, on the course \( l_1 \ldots l_7 \), the adapted pseudoinverse of the Jacobian matrix (relation (9) with \( r = 4 \)). The effect of applying this adapted pseudoinverse is felt only by the angles \( \theta_i, i = 5 \ldots 7 \), and not by the angles \( \theta_i, i = 1 \ldots 4 \), that will remain unchanged.

Using the values for \( \theta_i, i = 1 \ldots 4 \), calculated at the A4 step, and for \( \theta_i, i = 5 \ldots 7 \), calculated at the A6 step, the initial state is recomposed, after the first step generated on the entire course \( l_1 \ldots l_7 \). For the next step generation, using the known position (velocity) variations of the end-effector points E_1 and E_2, one proceeds as above. For the coupling of the lower branch \( l_5 \ldots l_7 \) one proceeds in the same way.

6. COMPUTER SIMULATION RESULTS

In this paper, the proposed method and algorithm for the kinematic simulation of a ramified planar redundant structure with ten elements is applied, using the MATLAB 6 program, in two cases aiming the planar manipulation of the segment \( E_1E_2 \):

- the segment \( E_1E_2 \) rotates with angle \( \pi \) while its middle point moves along a straight line.
- the segment \( E_1E_2 \) rotates with angle \( \pi \) while its middle point moves along an arc of ellipse.

In both presented cases, the arms are cooperating for the segment \( E_1E_2 \) rotation and translation. The computer simulation results are shown in the figures 4 and 5.
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Figure 5 - Segment $E_1E_2$ roto-translation with angle $\pi$ along a quarter of ellipse.

The simulation results appreciation is made qualitatively, observing step-by-step the successive positions occupied by the structure elements during the simulation progress, especially the segment $E_1E_2$ displacement. One notices the monotone variation of the angles of each structure element.

7. CONCLUSIONS

This work is the first authors’ approach to the kinematic study and simulation of the open ramified planar redundant structures. The use of the proposed method and algorithm offers the possibility to tackle kinematic structures similar to the human skeleton in order to study and simulate the simultaneous biped walking and arms cooperating work.

In this stage, the obtained simulation results prove that the proposed method and algorithm offers enough hopes for a satisfactory analyse of kinematic structures with two pairs of ramifications, “arms” and “legs”, which come closer to the human skeleton.

The analysis of the absolute errors in the generation of the imposed steps indicate that the errors decrease when small incremental steps (small velocities) for the points $E_1$ and $E_2$ are used. One observes that the errors increase in the vicinity of the singular positions, when the neighbouring elements tend to be collinear or superposed.

REFERENCES