THE DAMPING CAPACITY OF THE NANOSTRUCTURED MATERIALS FROM CARBON NANOTUBES

Veturia CHIROIU

Institute of Solid Mechanics, Bucharest, email: veturiachiroiu@yahoo.com

In this paper, the damping capacity of the composites reinforced by single walled carbon nanotubes is investigated. A thin stratified plate consisted of two viscoelastic polymer layers and a nano-adhesive layer reinforced by carbon nanotubes in low concentrations 1-3% by weight, is considered. We show that the frictional sliding at the nanotube-polymer interfaces have the potential to cause significant dissipation of energy.

Key words: damping capacity, nanocomposites, carbon nanotubes.

1. INTRODUCTION

By integrating nanotubes into traditional materials (epoxy matrices) it is possible to improve their damping capacity and ability to reduce vibrations, especially at high temperatures. Nanocomposites have a reduced weight and volume, in comparison to polymeric damping materials, and have a high damping capabilities. This is because they have a great applicability in aircraft, spacecraft, automobiles and sensors.

Since their discovery in 1991, the carbon nanotubes offer an interesting combination of light and stiff, of high strength and damping. Since the carbon nanotubes are so thin, they possess a big amount of surface area for the volume they occupy \( \frac{4}{d^2} \cdot \frac{m^2}{gm} \), giving them a great capacity to dissipate energy (Ko et al.[1]). Carbon nanotubes are up to 100 times stronger than steel at only a sixth the weight, and have attracted much research as potential to improve the strength of composite materials.

The theoretical and experimental results from literature have demonstrated that carbon nanotubes can be used to increase damping in composite structures. It is also important to ensure that the carbon nanotubes do not have a negative influence on the fatigue life and structural integrity on the composite. The principal experimental results used in this paper are furnished by El-Fattah Ali [2] and Saito [3]. This damping is related to the interfacial “slip-stick” friction between nanotubes and polymer. There is a direct advantage of interfacial sliding at the nanotube-polymer interfaces to enhance structural damping without changing the structural integrity (Koratkar [4–6]).

In this paper we consider a thin plate consisted of two viscoelastic polymer layers and a nano-adhesive layer uniformly reinforced by carbon nanotubes. We investigate the role of the carbon nanotubes in helping dampen vibrations of this structure subjected to bending vibrations.

2. THEORY

In this paper we try to find a class of constitutive laws for carbon nanotubes by applying the pseudospherical reduction method for a mechanical problem (Rogers and Schief [7],[8]). By this reduction the motion equations are associated to a pseudospherical surface \( \Sigma \) (with negative Gaussian curvature \( K \)).

If the ratio \( K/d^4 \), where \( d \) is the distance from the origin to the tangent plane at an arbitrary point is constant, we obtain a Tzitzeica surface (Tzitzeica [9], [10], Teodorescu et al [11]). The Tzitzeica surfaces are invariants under the group of centroaffine transformations, being analogues of spheres in affine differential geometry. At present, only a few achievements have been made for determining the parametrical
representation for a class of constitutive laws for which the motion equations attached to the material system can be associated to a pseudospherical surface. We consider that this novel approach may improve the estimation of structurally sensitive material properties. The bending theory of the plate is based on the references [12–18].

Let us consider a thin plate consisted of two viscoelastic polymer (p-phenelyene benzobisthiazole) layers and a nanoadhesive layer consisted by an epoxy matrix (polyhedral oligomeric silsesquioxane POSS from macromers epoxide group) uniformly reinforced by single walled carbon nanotubes in low concentrations 1-3% by weight (fig.2.1). A section through the nano-adhesive layer is represented in fig. 2.2. The elastic bonding in the nanoadhesive layer permits for frictional sliding between the carbon nanotubes and the epoxy. We show that the frictional sliding at the nanotube-polymer inside the adhesive, and the nanotube-polymer outside at the interfaces, has the potential to cause dissipation of energy. The nanotubes within the nanoadhesive layer can easily slide relative to each other. Only damping hysteresis resulting from dynamic loading conditions is considered. The loss elasticity modulus $E''$ is defined by the imaginary portion of the complex elasticity modulus \( \hat{E} = E - iE' \). The dissipation factor is defined by the ratio of the loss modulus to the modulus of elasticity \( D = \frac{E''}{E} \).

A single wall carbon nanotube (SWNT) can be described as a graphene sheet rolled into a cylindrical shape so that the structure is 1D with axial symmetry, and in general exhibiting a spiral configuration, called chirality. A single wall carbon nanotube is defined also by a cylindrical graphene sheet with a diameter of about 0.7–10.0 nm, though most of the observed SWNT have diameters <2 nm. If we neglect the two ends of a carbon nanotube and focus on the large aspect ratio of the cylinder the length/diameter ratio can be as large as $10^4 - 10^5$.

![Fig. 2.1. A plate of two stratified polymer layers and a standard adhesive (left), the same plate with a nano-adhesive layer (right).](image)

![Fig. 1.2. A section through the nanoadhesive layer. Circles refer to carbon nanotubes blended in POSS.](image)

The viscoelastic model described in this paper gives rise to energy dissipation as a loss of energy. The energy dissipated per cycle, $\Delta W$ is calculated from $\Delta W = \int_0^{2\pi / \omega} \sigma \varepsilon dt$. The energy loss is expressed in terms of a loss factor $Q^{-1} = \frac{1}{2\pi} \frac{\Delta W}{W_0} = \sin \delta$, where $W_0$ is the total vibrational energy stored in the structure.
Writing the constitutive law in term of the complex modulus \( \sigma = E\varepsilon \) we can calculate the phase angle \( \delta \) and the relaxation time \( \tau \). The decrement \( \delta \) is defined as the ratio of the energy loss per cycle, \( \Delta W \), to twice the total vibrational energy \( W_0 \), that is \( \delta = \frac{\Delta W}{2W_0} \).

Consider now the 1D problem of uniaxial deformation of a nonhomogeneous rod. We present in this section the pseudospherical reduction of the problem in the spirit of Rogers and Schief [7], [8]. The governing equations in a Lagrangian system of coordinates \((X,t)\) are written as

\[
\varepsilon = v_X, \quad \rho_0 v = \sigma_x.
\]  

(2.1)

The general constitutive law is given by

\[
\sigma = \sigma(\varepsilon, X).
\]  

(2.2)

Here, \( \sigma \) and \( \rho \) are the uniaxial stress and respectively, the density of the material, \( \varepsilon = \frac{\rho_0}{\rho} - 1 \) is the stretch, \( \rho_0 \) is the density of the material in the underformed state, and \( v(X,t) \) is the material velocity. In terms of the Eulerian coordinates \( x = x(X,t) \), we have

\[
dx = (\varepsilon + 1)dX = vd\tau,
\]  

(2.3)

so that

\[
\rho_0 dX = \rho dx - \rho v dt.
\]  

(2.4)

In (2.4), \( X \) corresponds to the particle function \( \psi \) of the Martin formulation. The independent variables are chosen to be \( \sigma \) and \( \psi \), and we suppose \( \rho_0 = 1 \). In this case we obtain the Monge–Ampère equation

\[
\frac{\xi}{\xi_{\sigma\sigma}} \frac{\xi}{\xi_{\sigma\psi}} - \frac{\xi^2}{\xi_{\sigma\psi}} = \xi_{\sigma},
\]  

(2.5)

where

\[
t = \xi_{\sigma}, \quad v = \xi_{\psi},
\]

\[
dx = \xi_{\psi}\xi_{\sigma\sigma} + (\xi_{\sigma\sigma\psi} + \psi) d\psi,
\]  

(2.6)

\[0 < |\xi_{\sigma\sigma\psi}| < \infty .\]

If a solution \( \xi(\sigma,\psi) \) of this equation is specified, then the particle trajectories are calculated from

\[
x = \int[\xi_{\sigma\sigma\psi} + (\xi_{\sigma\sigma\psi} + \psi) d\psi], \quad t = \xi_{\sigma},
\]  

(2.7)

in terms of \( \sigma \), for \( \psi = \text{const.} \). By solving (2.7), the solution \( \sigma(\psi,t) \) is obtained, and the original solution of (2.1)–(2.2) is parametrically determined in terms of the Lagrangian variables

\[
x = x(\psi,t), \quad v = v(\psi,t), \quad \sigma = \sigma(\psi,t).
\]  

(2.8)

To show the geometric connection to this problem, let us consider a surface \( \Sigma \) in \( \mathbb{R}^3 \) written the Monge parametrisation

\[
r = xe_1 + ye_2 + z(x,y)e_3,
\]  

(2.9)

where \( r = r(x,y,z) \) the position vector of a point \( P \) on the surface.

The first and second fundamental forms are defined as

\[
I = Edx^2 + 2Fdx^2 + Gdy^2 = (1 + z_x)dx^2 + 2z_xz_y dy + (1 + z_y)^2 dy^2,
\]  

\[
II = \left(\begin{array}{cc}
a_{11} & a_{12} \\
a_{21} & a_{22} \end{array}\right)
\]
The Gaussian curvature of $\Sigma$ is

$$K = \frac{eg - f^2}{EG - F^2} = \frac{z_{xx}z_{yy} - z_{xy}^2}{(1 + z_x^2 + z_y^2)^2}.$$  \hfill (2.11)

If $\Sigma$ is a hyperbolic surface, then total curvature is negative and the asymptotic lines on $\Sigma$ may be taken as parametric curves. Let us introduce the same independent variables as before, $\sigma$ and $\psi$

$$\sigma = z_x, \quad \psi = z_y,$$  \hfill (2.12)

and the dependent variable $\xi$

$$\xi_\sigma = x, \quad \xi_\psi = y.$$  \hfill (2.13)

Therefore, we have

$$\xi_{\sigma\sigma} = \frac{z_{yy}}{z_{xx}z_{yy} - z_{xy}^2}, \quad \xi_{\psi\psi} = \frac{z_{xx}}{z_{xx}z_{yy} - z_{xy}^2}, \quad \xi_{\sigma\psi} = \frac{z_{xy}}{z_{xx}z_{yy} - z_{xy}^2}.$$  \hfill (2.14)

The Gaussian curvature (2.10) yields

$$K = \frac{1}{(1 + \sigma^2 + \psi^2)^2 (\xi_{\sigma\sigma}\xi_{\psi\psi} - \xi_{\sigma\psi}^2)}.$$  \hfill (2.15)

The Gaussian curvature may be set into correspondence with the Martin’s Monge–Ampère equation (2.5) by

$$\varepsilon_\sigma = \frac{1}{K(1 + \sigma^2 + \psi^2)^2},$$  \hfill (2.16)

and

$$K = \frac{A^2}{(1 + \sigma^2 + X^2)^2}.$$  \hfill (2.17)

where $A^2 = \frac{\partial \sigma}{\partial \varepsilon} \bigg|_X$, with $A$ the Lagrangian wave velocity. The surface $\Sigma$ is restricted to be pseudospherical, that is

$$K = -\frac{1}{A^2}, \quad a = \text{const.}$$  \hfill (2.18)

In this case the relation (2.17) gives

$$\frac{\partial^2 \sigma}{\partial \varepsilon^2} \bigg|_X = \frac{2}{a^2} (1 + \sigma^2 + X^2) \sigma \frac{\partial \sigma}{\partial \varepsilon} \bigg|_X > 0, \quad \sigma > 0.$$  \hfill (2.19)

Integrating (2.19) we have

$$\varepsilon = \frac{a^2}{2(1 + X^2)^{3/2}} \left[ \arctan(\frac{\sigma}{1 + X^2}) + \frac{\sqrt{1 + X^2}}{1 + \sigma^2 + X^2} \right] + \alpha(X),$$  \hfill (2.20)
In particular, let us introduce into (2.19) the stress representation

\[ \sigma = \sqrt{1 + X^2} \tan \left( \frac{\sqrt{1 + X^2}}{a} (c - c_0) \right), \]  

(2.21)

In this case we obtain

\[ \varepsilon = \frac{a^2}{2(1 + X^2)} \left[ \frac{c - c_0}{a} + \frac{1}{\sqrt{1 + X^2}} \sin \left( \frac{2\sqrt{1 + X^2}}{a} (c - c_0) \right) \right]. \]  

(2.22)

Thus, relations (2.21) and (2.22) represent a parametric representation for the constitutive laws \( \sigma = \sigma(\varepsilon, X) \), for which the equations (2.1) are associated to a pseudospherical surface \( \Sigma \).

These equations lead to

\[ \sigma_{xx} = \varepsilon_n. \]  

(2.23)

Using (2.16), we have

\[ \sigma_{xx} = \left[ \frac{a^2}{(1 + \varepsilon_n^2 + X^2)} \sigma_n \right]. \]  

(2.23)

The equation (2.23) has a solitonic behavior and admits soliton solutions (Munteanu and Donescu 2002, 2004). These solutions known as solitons have the form of localized functions that conserve their properties even after interaction among them, and then act somewhat like particles.

The equation (2.23) like others remarkable equations (Korteweg and de Vries, Burgers, sine-Gordon, Schrödinger, etc.) has interesting properties: an infinite number of local conserved quantities, an infinite number of exact solutions expressed in terms of the Jacobi elliptic functions (cnoidal solutions) or the hyperbolic functions (solitonic solutions or solitons), and the simple formulae for nonlinear superposition of explicit solutions.

Such equations were considered integrable or more accurately, exactly solvable. Given a nonlinear equation, it is natural to ask whether it is integrable, or it admits the exact solutions or solitons, whether its solutions are stable or not. This question is still open, and efforts are made for collecting the main results concerning the analysis of nonlinear equations such as (2.23) (Munteanu and Donescu [19]). Equation (2.23) is used in this article to model the constitutive behavior of carbon nanotubes, due to the perfect agreements between this law and the available experimental data Gao et al. [20], Yu et al. [21], Yakobson et al. [22], Whitesides and Alivisatos [23] and Hernandez et al. [24].

3. RESULTS

The plate with and without nanotube reinforcement in the adhesive layer. with 2% and 3% single-walled carbon nanotubes (SWNCT) by weight, are dynamically strained in the bending mode (0-50 Hz frequency range). Figs. 3.1 and 3.2 show the stress–strain loading and unloading constitutive laws for 2% and respectively for 3% SWNCT by weight, determined by applying the pseudospherical reduction method.

The results for the plate with standard adhesive layer (POSS) is also shown for comparison. Note that the modulus of the plate with nanoadhesive layer is about twice that of the baseline plate.

Fig. 3.3 shows the loss factor for both plates with the standard adhesive layer as well as the nano-adhesive layer (with 2% and 3% SWNTC by weight). Even though the epoxy has a very high loss factor (~1), the carbon nanotubes integrated into the adhesive layer cause a significant increase in damping, particularly at the higher frequencies.

This energy dissipation results from interfacial friction as well as the sliding that occurs within the adhesive layer between the epoxy and carbon nanotubes.
Fig. 3.1. The stress–strain loading and unloading constitutive laws for both plates with the standard adhesive layer and the nanoadhesive layer with 2% SWCNT by weight.

Fig. 3.2. The stress–strain loading and unloading constitutive laws for both plates with the standard adhesive layer and the nanoadhesive layer with 3% SWCNT by weight.
4. CONCLUSIONS

Carbon nanotubes are up to 100 times stronger than steel at only a sixth the weight, and have attracted much research as potential additives to improve the strength of composite materials. The nanocomposites show great potential for a variety of applications in aircraft, spacecraft, automobiles, and even sensors for dynamical systems.

Frictional sliding of the carbon nanotubes with the polymer and with the adhesive have the potential to cause significant dissipation of energy. Since the nanotubes are so tiny, they altogether possess an incredible amount of surface area for the volume they occupy, giving them a great capacity to dissipate energy.

In this paper, the damping capacity of the composites reinforced by SWCNT is investigated. For a thin stratified plate, consisted of two viscoelastic polymer layers and a nanoadhesive layer reinforced by carbon nanotubes in low concentrations 2% and 3% by weight, we show that the frictional sliding at the nanotube-polymer interfaces have the potential to cause significant dissipation of energy.

ACKNOWLEDGEMENT. The authors acknowledge the financial support of the National University Research Council (NURC-CNCSIS) Romania, Grant nr. 33344/2005, code 152/2003, Grant nr. 27664/2005, code160/2005 and Grant CEEX 1531/2006.

REFERENCES